THIN-LAYER FLOW OF NONLINEAR ELASTOVISCOUS FLUID IN A FIELD OF CENTRIFUGAL FORCES

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UDC 532.135

The flow is considered of an elastoviscous fluid whose deformation state is expressed by means of kinematic matrices in the form of a thin layer between two rotating coaxial cones with the same angular velocity.

The rheological equation of state for a nonlinear elastoviscous fluid [1-4]

$$\sigma = -pI + \mu_{eff}B_1 + \frac{1}{2}\beta_1 B_2 \tag{1}$$

has been widely studied lately.

The kinematic matrices $B_{1,2}$ depend on the gradients of velocities, accelerations and higher time derivatives of velocity; μ_{eff}, β_1 are functions of the invariants $B_{1,2}$.

For an incompressible medium the matrix B_1 is equivalent to the classical velocity tensor of shearing strain for a Newtonian fluid.

The coefficient of effective viscosity can be written as [5]

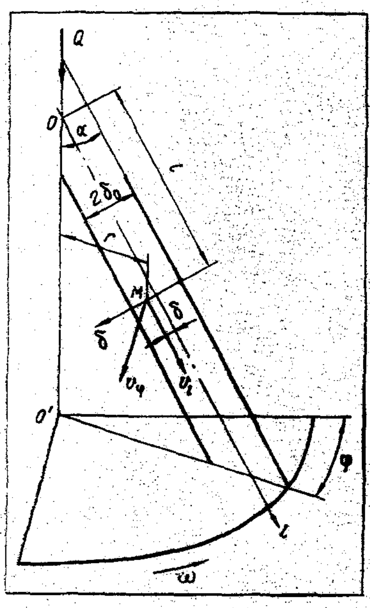

Fig. 1

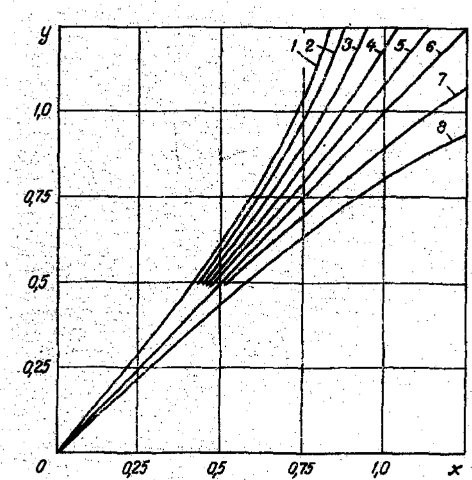

Fig. 2

Fig. 1. Flow diagram.

Fig. 2. Graphs of y = f(x): 1) n = 0.1; 2) 0.2; 3) 0.4; 4) 0.6; 5) 0.8; 6) 1.0; 7) 1.4; 8) 1.9.

Kirov Institute of Chemical Engineering, Kazan. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 2, pp. 266-271, February, 1974. Original article submitted May 16, 1973.

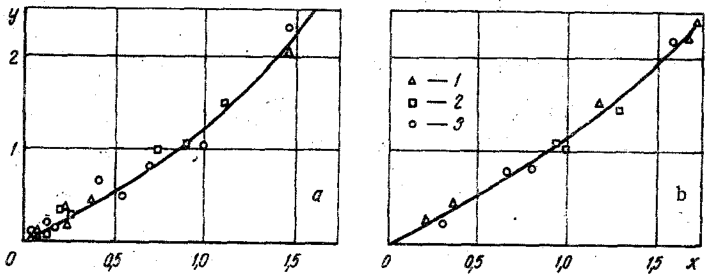

Fig. 3. The relation y = f(x): a) polyacrylamide solution at K = 15 dyne · sec^n/cm^2, n = 0.56; b) the same at K = 3.15 dyne · sec^n/cm^2, n = 0.66. The points show experimental results (1) flat nozzle of 80 mm diam.; 2α flat nozzle of 100 mm diam.; 3) conical nozzle of 80 mm diam., α = 60°). Continuous curves represent theoretical data.

$$\mu_{eff} = KE^{n-1}. \tag{2}$$

The coefficient β_1 determines the magnitude of the expansion (or compression) normal stresses along the stream lines. It was shown in [6-8] that β_1 is proportional to the square of viscosity and is given by the formula

$$\beta_1 = \frac{1}{G_0}\mu_{eff}^2. \tag{3}$$

The solution is given below of the flow of nonlinear elastoviscous fluid whose rheological state equation is governed by Eq. (1) in a field of centrifugal forces.

Let there be a nonlinear elastoviscous flow in the form of a thin layer between two rotating coaxial cones with the same angular velocity (Fig. 1). The generators of the cones are parallel. It is assumed that: 1) the flow is steady and the fluid incompressible, 2) the distance between the cones is considerably smaller than between the generators, 3) the flow is symmetrical relative to the axis of rotation, 4) the angular velocity is sufficiently high so that gravitation force can be ignored.

The fluid flow is considered in a special coordinate system l, φ, δ, rigidly fixed to the cone. The introduced system is orthogonal, its Lamé coefficients being: $H_l = 1$, $H_\varphi = r - \delta\cos\alpha$, $H_\delta = 1$. By considering $\delta/l << 1$ one can write $H_\varphi = r$.

The differential equations in the adopted coordinate system with the assumptions taken into account are given by:

$$-\frac{\partial p}{\partial l} + \rho F_l + \frac{K\partial}{\partial\delta}\left[E^{n-1}\frac{\partial v_l}{\partial\delta}\right] - \frac{K^2}{G_0}\frac{\partial}{\partial l}\left[E^{n-1}\frac{\partial v_l}{\partial\delta}\right]^2 - \frac{K^2\sin\alpha}{G_0 r}E^{2(n-1)}\left[\left(\frac{\partial v_l}{\partial\delta}\right)^2 - \left(\frac{\partial v_\varphi}{\partial\delta}\right)^2\right] = 0, \tag{4}$$

$$\rho F_\varphi + \frac{K\partial}{\partial\delta}\left[E^{n-1}\frac{\partial v_\varphi}{\partial\delta}\right] - \frac{K^2}{G_0 r^2}\frac{\partial}{\partial l}\left[r^2E^{2(n-1)}\frac{\partial v_l}{\partial\delta}\frac{\partial v_\varphi}{\partial\delta}\right] = 0, \tag{5}$$

$$-\frac{\partial p}{\partial\delta} + \rho F_\delta + \frac{K}{r}\frac{\partial}{\partial l}\left[rE^{n-1}\frac{\partial v_l}{\partial\delta}\right] = 0,\quad E = \sqrt{\left(\frac{\partial v_l}{\partial\delta}\right)^2 + \left(\frac{\partial v_\varphi}{\partial\delta}\right)^2}. \tag{6}$$

The boundary conditions are:

$$\text{for } \delta = \delta_0 \quad v_l = v_\varphi = 0,$$
$$\text{for } \delta = 0 \quad \frac{\partial v_l}{\partial\delta} = \frac{\partial v_\varphi}{\partial\delta} = 0. \tag{7}$$

The body forces consist of the Coriolis and the centrifugal forces:

$$F_l = (r\omega^2 - 2\omega v_\varphi)\sin\alpha,$$
$$F_\varphi = 2\omega v_l\sin\alpha, \tag{8}$$
$$F_\delta = (-r\omega^2 + 2\omega v_\varphi)\cos\alpha.$$

To find the solution it is assumed that the original functions v_l, v_φ, p are polynomials in the parameter δ. By virtue of the flow symmetry they depend only on l and δ. Their form is [9,10,11]:

$$v_l = v_l(l,\ \delta) = a_l(l) + b_l(l)\delta + c_l(l)\delta^{\frac{n+1}{n}},$$

$$v_\varphi = v_\varphi(l,\ \delta) = a_\varphi(l) + b_\varphi(l)\delta + c_\varphi(l)\delta^{\frac{n+1}{n}},$$

$$p = p_1(l) + p_2(l)\delta. \tag{9}$$

By using the boundary conditions (8) the relations (9) can be replaced by:

$$v_l = c_l(l)\left(\delta^{\frac{n+1}{n}} - \delta_0^{\frac{n+1}{n}}\right),$$

$$v_\varphi = c_\varphi(l)\left(\delta^{\frac{n+1}{n}} - \delta_0^{\frac{n+1}{n}}\right). \tag{10}$$

To determine v_l the relation is used

$$Q = 2\int v_l dS.$$

By inserting the value of v_l from (10) an expression is obtained for the meridional velocity,

$$v_l = \frac{Q}{4\pi r\delta_0^{\frac{2n+1}{n}}}\left(\frac{2n+1}{n+1}\right)\left(\delta_0^{\frac{n+1}{n}} - \delta^{\frac{n+1}{n}}\right). \tag{11}$$

Equation (5) is used to determine v_φ. By using the Karman—Pohlhausen method one obtains the system of equations

$$\int_0^{\delta_0} \rho F_\varphi r d\delta + \int_0^{\delta_0} K\frac{\partial}{\partial\delta}\left[E^{n-1}\frac{\partial v_\varphi}{\partial\delta}\right] r d\delta - \int_0^{\delta_0}\frac{K^2}{G_0 r^2}\frac{\partial}{\partial l}\left[r^2E^{2(n-1)}\frac{\partial v_l}{\partial\delta}\frac{\partial v_\varphi}{\partial\delta}\right] r d\delta = 0,$$

$$\int_0^{\delta_0}\rho F_\varphi v_l r d\delta + \int_0^{\delta_0} K v_l\frac{\partial}{\partial\delta}\left[E^{n-1}\frac{\partial v_\varphi}{\partial\delta}\right] r d\delta$$

$$-\int_0^{\delta_0}\frac{K^2 v_l}{G_0 r^2}\frac{\partial}{\partial l}\left[r^2E^{2(n-1)}\frac{\partial v_l}{\partial\delta}\frac{\partial v_\varphi}{\partial\delta}\right] r d\delta = 0. \tag{12}$$

By evaluating the integrals in Eqs. (12) using the boundary conditions (7) as well as the constancy of the flow rate per second and by eliminating from these equations $\partial c_\varphi/\partial l$, one obtains after some transformations the following expression for the sought c_φ:

$$c_\varphi = -\frac{\rho\omega\sin\alpha\left(\frac{Q}{4\pi}\right)}{\delta_0 rK\left(\frac{n+1}{2n+1}\right)\left(\frac{n+1}{n}\right)^n\left(\frac{3n+2}{n+4}\right)\left(c_l^2 + c_\varphi^2\right)^{\frac{n-1}{2}}}. \tag{13}$$

Using the notation

$$y = -\frac{c_\varphi}{c_l},\quad x = \frac{\rho\omega\sin\alpha\, r^{n-1}\delta_0^{2n}}{K\left(\frac{2n+1}{n+1}\right)^{n-1}\left(\frac{n+1}{n}\right)^n\left(\frac{3n+2}{n+4}\right)\left(\frac{Q}{4\pi}\right)^{n-1}},$$

one obtains from (13)

$$x = y(1+y^2)^{\frac{n-1}{2}}. \tag{14}$$

Using Fig. 2 one can solve graphically the relation (14) for y if x is known.

An expression for p is found similarly as in [11] by ignoring the term $K/r\ \partial/\partial l\ [rE^{n-1}\partial v_l/\partial\delta]$ in (6) in view of its smallness compared to ρF_δ.

Averaging over the thickness one obtains an expression for average velocities from (10) and (11):

$$v_{l\text{ave}} = \frac{Q}{4\pi l \sin\alpha\delta_0}, \quad v_{\varphi\text{ave}} = \frac{yQ}{4\pi l \sin\alpha\delta_0}. \tag{15}$$

It follows from (15) that for a given state $v_{l\text{ ave}}$ and $v_{\varphi\text{ ave}}$ are linearly related.

The values of the greatest possible flow rate are now determined such that there is no choking. To this end it is assumed that in the first approximation $v_\varphi = 0$, $\partial p/\partial l << \rho F_l$.

Equation (4) then becomes

$$\rho F_l + K\frac{\partial}{\partial\delta}\left(\frac{\partial v_l}{\partial\delta}\right)^n - \frac{K^2}{G_0}\frac{\partial}{\partial l}\left(\frac{\partial v_l}{\partial\delta}\right)^{2n} - \frac{K^2}{G_0 l}\left(\frac{\partial v_l}{\partial\delta}\right)^{2n} = 0. \tag{16}$$

The boundary conditions are:

$$\text{for } \delta = \pm\delta_0 \qquad v_l = 0,$$
$$\text{for } \delta = 0 \qquad \frac{\partial v_l}{\partial\delta} = 0. \tag{17}$$

The profile is given as

$$v_l = v_{l\max}\left(1 - \left|\frac{\delta}{\delta_0}\right|^{\frac{n+1}{n}}\right), \tag{18}$$

then the relation between $v_{l\max}$ and $v_{l\text{ave}}$ can be written as

$$v_{l\max} = v_{l\text{ave}}\left(\frac{2n+1}{n+1}\right). \tag{19}$$

One obtains from Eq. (16), similarly as in (12), the equation

$$\int_0^{\delta_0} \rho F_l r d\delta + \int_0^{\delta_0} K\frac{\partial}{\partial\delta}\left(\frac{\partial v_l}{\partial\delta}\right)^n r d\delta - \int_0^{\delta_0}\frac{K^2}{G_0}\frac{\partial}{\partial l}\left(\frac{\partial v_l}{\partial\delta}\right)^{2n} r d\delta$$
$$- \int_0^{\delta_0}\frac{K^2}{G_0 l}\left(\frac{\partial v_l}{\partial\delta}\right)^{2n} r d\delta = 0. \tag{20}$$

By evaluating the integrals appearing in Eq. (20) and using the boundary conditions (17) and (19) one obtains

$$(v_l\text{ave})^{-2n} - (v_l\text{ave})^{-n}\frac{K\left(\frac{2n+1}{n}\right)^n}{\rho F_l \delta_0^{n+1}} - \frac{2K^2(1-n)\left(\frac{2n+1}{n}\right)^{2n}}{3\rho G_0 F_l \delta_0^{2n} l} = 0. \tag{21}$$

Equation (21) can easily be solved and one is thus able to calculate the value of the flow rate per second for entry conditions:

$$v_{l\text{ave}} = \frac{1}{\left[\frac{K\left(\frac{2n+1}{n}\right)^n}{2\rho F_l \delta_0^{n+1}}\right]^{\frac{1}{n}}\left(1+\sqrt{1+A}\right)^{\frac{1}{n}}}. \tag{22}$$

In the above $A = \frac{8(1-n)\rho\delta_0^2\omega^2\sin\alpha}{3G_0}$ is the elasticity parameter.

It can be seen from (22) that with the elasticity parameter increasing the value of the average meridional velocity is reduced. For $G_0 = 400$, 200, 100 dyne/cm^2, $n = 0.5$, $\omega = 100$ sec^{-1}, $\rho = 1.02$ g/cm^3, $\delta_0 = 0.1$ cm, $\alpha = 60°$, one has

$$\frac{v_l\text{ave}_{|G_0=400,\ 200,\ 100}}{v_l\text{ave}_{|G_0=\infty}} \doteq 0.87;\ 0.78;\ 0.66.$$

Experiments were carried out to verify these results. Details of the experiments were described in [11]. Aqueous solutions of polyacrylamide (PAAM) were taken as examples. A capillary viscometer of constant pressure was used to determine the rheological constants.

The experiments were carried out for flat and conical nozzles. The experimental results are shown in Fig. 3 where it can be seen that the experimental points are convincingly grouped near the theoretical curve and it can thus be concluded that the rheological state equation used by us can be applied.

NOTATION

p	is the hydrostatic pressure;
I	is the identity matrix;
K, n	are the rheological fluid constants;
E	is the second invariant of strain rate tensor;
G_0	is the initial modulus of high elasticity;
σ	is the stress tensor;
H_1, H_2, H_3	are the Lamé coefficients;
ρ	is the fluid density;
l, φ, δ,	are the cone generator, length, distance from axis to fluid particle M;
r	is the distance between surface and axis of revolution;
2α	is the cone vertex angle;
F_l, F_φ, F_δ	are the projections of body forces in directions l, φ, and δ respectively;
v_l	is the meridional velocity;
v_φ	is the lag velocity;
$2\delta_0$	is the distance between cones;
ω	is the angular velocity of revolution of conic surfaces;
Q	is the fluid flow rate per sec.;
S	is the cross section areas between cones.

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